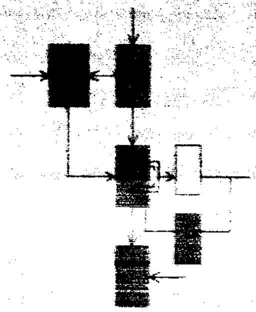
May, 1966

REPORT ESL-R-270
M.I.T. PROJECT DSR 6152
NASA Research Grant
NsG-496 (Part)



A HYBRID GRAPHICAL DISPLAY TECHNIQUE

Huber L. Graham

Electronic Systems Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY, CAMBRIDGE, MASSACHUSETTS 02139

Department of Electrical Engineering

May, 1966 Report ESL-R-270 Copy 8

A HYBRID GRAPHICAL DISPLAY TECHNIQUE

by

Huber L. Graham

The preparation and publication of this report, including the research on which it is based, was sponsored under a grant to the Electronic Systems Laboratory, Massachusetts Institute of Technology, Project DSR No. 6152. This grant is being administered as part of the National Aeronautics and Space Administration Research Grant No. NsG-496 (Part). This report is published for information purposes only and does not represent recommendations or conclusions of the sponsoring agency.

Electronic Systems Laboratory
Department of Electrical Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

ABSTRACT

An on-line graphical display technique and an experimental system prototype employing this technique are discussed. Unlike some methods, which display graphical data by point-plotting or by piecewise-linear segments, this technique employs a sequence of curved segments. This approach results in compact storage of the digital commands that describe a complex curve at the expense of some computing time necessary to establish these commands.

The prototype system, which is based on this technique, is relatively uncomplicated and inexpensive; therefore, it is suited for use at the remote consoles of a time-shared computer facility.

AUTHOR

ACKNOWLEDGMENT

The material presented in this report is based on a thesis submitted in May, 1966 in partial fulfillment of the requirements for the Master of Science Degree in Electrical Engineering.

The author wishes to express his appreciation to Professor Michael L. Dertouzos for help and suggestions in the supervising of this work.

This research was sponsored in part by the National Aeronautics and Space Administration under contract number NsG-496 (Part).

Work reported herein was supported (in part) by Project MAC, an M.I.T. research program sponsored by the Advanced Research Projects Agency, Department of Defense, under Office of Naval Research Contract Number Nonr-4102(01).

CONTENTS

LIST OF FIGURES

LIST OF FIGURES (Cont'd.)

I. INTRODUCTION

Graphical displays are gaining unquestioned importance in the growing communication between people and digital computers. These displays are vital in a number of diverse fields where rapid analysis of graphical data is desired. This research describes an on-line graphical display technique, first proposed by Professor M. L. Dertouzos,[1*] and an experimental system prototype, which has the ability to display continuous curves with relatively few computer commands. The prototype has proven to be of considerable use in conjunction with a time-shared computer facility where a rapid man-machine interchange is desired.

---

*Superscripts refer to numbered items in the Bibliography

II. GENERAL CONCEPTS

A block diagram of the basic elements used in this technique is shown in Fig. 1. The visual output device is an x-y graphical display,

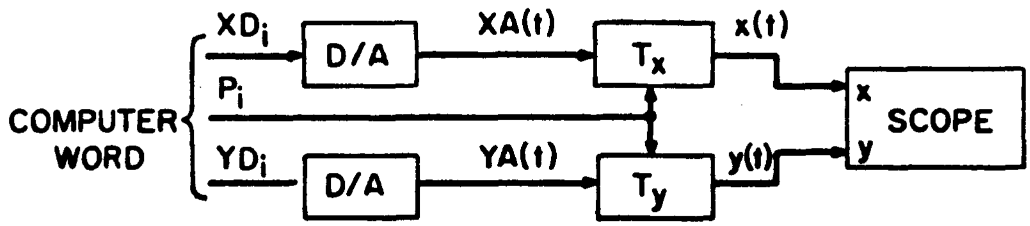


Fig. 1 Basic Configuration

such as a storage cathode-ray tube, driven by waveforms $x(t)$ and $y(t)$ which are continuous functions of time. These waveforms are the outputs of digital-to-analog converters which convert the coordinate values, XD_i and YD_i supplied by the computer, into analog form. A third part of the computer output word, P_i, controls the parameters of networks T_x and T_y. Changes of these parameters with each successive computer word is the central theme of the display technique.

Assume that at time t_k the linear networks have attained steady-state. As a consequence, the outputs of T_x and T_y will be $x(t_k) = XA(t_k)$ and $y(t_k) = YA(t_k)$, respectively. Suppose that the set of coordinates, supplied to the D/A converters at $t = t_k$, changes the value of XA by ΔX and YA by ΔY. The signals XA(t) and YA(t) will then be

$$XA(t) = XA(t_k) + \Delta X U_{-1}(t-t_k) \qquad (1)$$

$$YA(t) = YA(t_k) + \Delta Y U_{-1}(t-t_k) \qquad (2)$$

for $t \geq t_k$. Signals x(t) and y(t) will then be

$$x(t) = XA(t_k) + \Delta X T_x(t-t_k) \qquad (3)$$

$$y(t) = YA(t_k) + \Delta Y T_y(t-t_k) \qquad (4)$$

for $t \geq t_k$. Since T_x and T_y are constrained to have unity steady-state gain, the final values of x(t) and y(t) at time, t_{k+1}, where $t_{k+1}-t_k$ is much greater than the time constants of T_x and T_y, will be

$$x(t_{k+1}) = XA(t_k) + \Delta X \qquad (5)$$

$$y(t_{k+1}) = YA(t_k) + \Delta Y \qquad (6)$$

What is of interest, however, is the trajectory $f(x, y) = 0$ (obtained by eliminating time in (3) and (4)) followed by the display from point $(x(t_k), y(t_k))$ to point $(x(t_{k+1}), y(t_{k+1}))$. This, of course, will depend upon the nature of networks T_x and T_y and the way in which their internal parameters are controlled by P_i. For instance, in the special case where T_x is identical to T_y, Eqs. 3 and 4 give the following trajectory

$$y = YA(t_k) + \frac{\Delta Y}{\Delta X}(x - XA(t_k)) \qquad (7)$$

which is, as expected, a straight line. In the general case, when T_x and T_y are not identical, the trajectory will be some type of curve segment dependent upon the characteristics of T_x and T_y. It is desirable that T_x and T_y be properly chosen, so that a large family of curve segments will be available through appropriate control of parameters with P_i.

Assume for the moment that realizations of T_x and T_y exist which, as their parameters are varied, yield a large family of segments. The ability to synthesize large, complex curves from a relatively small number of these curve segments would then follow, making possible the display of that curve with a small number of computer commands. The problem of synthesizing an adequate realization of T_x and T_y, that is, a realization that yields a family of curve segments adequate for matching a large class of complex curves, is explored in the following section.

III. DESIGN STAGE

A. REALIZATIONS OF T_x AND T_y

As an initial step in developing an adequate realization for T_x and T_y, the advantages and disadvantages of a particularly simple solution will be discussed. This approach should shed some light on the desired characteristics of a good realization.

Let T_x and T_y have step responses which are rising exponentials, i.e.,

$$T_x(t) = [1-e^{-\sigma_x t}]\, U_{-1}(t) \tag{8}$$

$$T_y(t) = [1-e^{-\sigma_y t}]\, U_{-1}(t) \tag{9}$$

where σ_x and σ_y are positive and $U_{-1}(t)$ is the unit step. Then, from (3) and (4), if $XA(t_k) = X_0$, $YA(t_k) = Y_0$, $\Delta X = X_1 - X_0$, and $\Delta Y = Y_1 - Y_0$, the signals $x(t)$ and $y(t)$ will be

$$x(t) = [X_1 + (X_0 - X_1)e^{-\sigma_x(t-t_k)}]\, U_{-1}(t-t_k) \tag{10}$$

$$y(t) = [Y_1 + (Y_0 - Y_1)e^{-\sigma_y(t-t_k)}]\, U_{-1}(t-t_k) \tag{11}$$

for $t \geq t_k$. Eliminating time between Eqs. 10 and 11 give the trajectory between (X_0, Y_0) and (X_1, Y_1) as

$$y = Y_1 + (Y_0 - Y_1)\left(\frac{x-X_1}{X_0-X_1}\right)^{\sigma_y/\sigma_x} . \tag{12}$$

By varying the ratio of natural frequencies, σ_y/σ_x, a family of curve segments is obtained. Various members of this family have been plotted in Fig. 2 for $X_0 = Y_0 = 0$ and $X_1 = Y_1 = 1$. As shown, it is possible to obtain a large, well spaced family of curves by properly varying this parameter. This set of curve segments, however, has one major disadvantage. The slope at any point of a given segment is

$$\frac{dx}{dy} = \frac{Y_0 - Y_1}{X_0 - X_1} \cdot \frac{\sigma_y}{\sigma_x}\left(\frac{x - X_1}{X_0 - X_1}\right)^{\left(\frac{\sigma_y}{\sigma_x} - 1\right)} \tag{13}$$

which gives the final slope of a segment as

$$\left.\frac{dy}{dx}\right|_{x=X_1} = \begin{cases} 0 & \text{for } \sigma_y > \sigma_x \\ (Y_0 - Y_1)/X_0 - X_1) & \text{for } \sigma_y = \sigma_x \\ \infty & \text{for } \sigma_y < \sigma_x \end{cases} \tag{14}$$

Thus the final slope is either zero or infinity except for the special case when the segment is a straight line. The ability to connect two segments with a desired slope does not exist. This imposes the severe limitation of slope discontinuities on the displayed curves.

In retrospect, there appear to be three major characteristics of a good realization:

(1) It should provide a large and varied family of segments.

(2) It should have the ability to match slopes of connecting segments.

(3) It should have a configuration which can be reliably implemented.

Conditions (1) and (2) imply that the realization should have, besides steady-state coordinate, at least two degrees of freedom; one degree

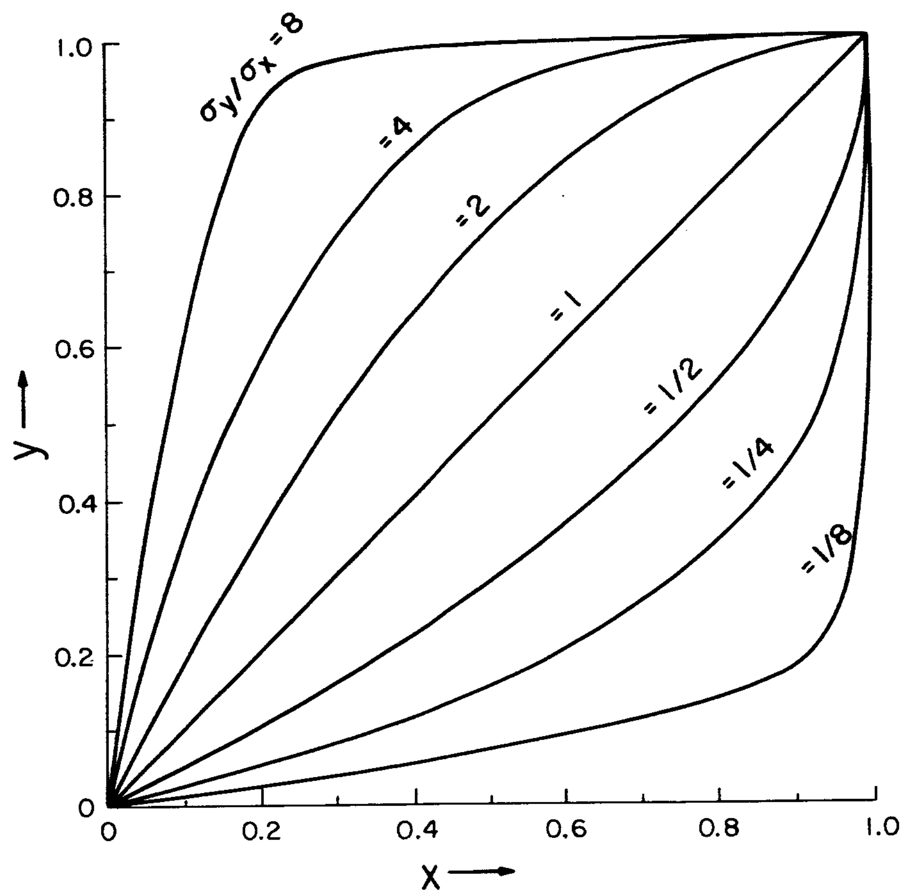


Fig. 2 Family of Curves from Simple Exponential Realization Obtained

to specify either initial or final slope, and a second degree to yield a family of segments having that slope. On this basis, two alternative realizations of T_x and T_y were investigated. To help satisfy condition (3), exponential time functions were used in both realizations.

B. POLYNOMIAL REALIZATION

Let $T_x(t)$ and $T_y(t)$ be of the form

$$T_x(t) = [1-e^{-\sigma t}]\ U\text{-}1(t) \tag{15}$$

$$T_y(t) = [\alpha+\beta e^{-\sigma t}+\gamma e^{-2\sigma t}+\delta e^{-3\sigma t}]\ U\text{-}1(t) \tag{16}$$

where α, β, γ, and δ are real constants. α is constrained to be the final value of y, in this case 1, and $\alpha+\beta+\gamma+\delta$ is constrained to be the initial value, in this case 0. A change in coordinates from (X_0, Y_0) to (X_1, Y_1) at time t_k give x(t) and y(t) as

$$x(t) = [X_1+(X_0-X_1)e^{-\sigma(t-t_k)}]\ U_{-1}(t-t_k) \tag{17}$$

$$y(t) = [Y_1+\beta e^{-\sigma(t-t_k)}+\gamma e^{-2\sigma(t-t_k)}+\delta e^{-3\sigma(t-t_k)}]\ U_{-1}(t-t_k) \tag{18}$$

for $t \geq t_k$, where $(\beta+\gamma+\delta)$ is constrained to be (Y_0-Y_1). The initial slope of this trajectory is

$$\left.\frac{dy}{dx}\right|_{x=X_0} = \frac{\beta+2\gamma+3\delta}{X_0-X_1} \tag{19}$$

and the resulting trajectory is a third-order polynomial of the form

$$y(x) = Ax^3+Bx^2+Cx+D \tag{20}$$

If the coefficients A, B, C, and D are constrained to have two degrees of freedom, the initial slope, m_0, and $Q = \gamma + \delta$, they become

$$A = (Q-Y_c+m_0X_c)/X_c^3 \tag{21}$$

$$B = (2Q+m_0X_c-Y_c)/X_c^2-3X_1(Q-Y_c+m_0X_c)/X_c^3 \tag{22}$$

$$C = (Q+Y_c)/X_c-2X_1(2Q+m_0X_c-Y_c)/X_c^2+3X_1^3(Q-Y_c+m_0X_c)X_c^3 \tag{23}$$

$$D = Y_1-X_1(Q+Y_c)/X_c+X_1^2(2Q+m_0X_c-Y_c)/X_c^2-X_1^3(Q-Y_c+m_0X_c)/X_c^3 \tag{24}$$

where $X_c = X_1-X_0$ and $Y_c = Y_1-Y_0$. A sampling of the curves which are generated by varying m_0 and Q is shown in Figs. 3 and 4. Using networks T_x and T_y of this type the initial slope of a curve segment can be specified to match the final slope of the previous segment, and a good fit to the "ideal" curve can then be obtained by varying Q. When such a good fit is found (by the computer), the desired values of α, β, γ, and δ are determined and supplied through P_i. This realization can match a large family of curves with no slope discontinuity. Moreover, more degrees of freedom are available by varying γ and δ independently instead of attempting to match slopes. In this manner any trajectory of the

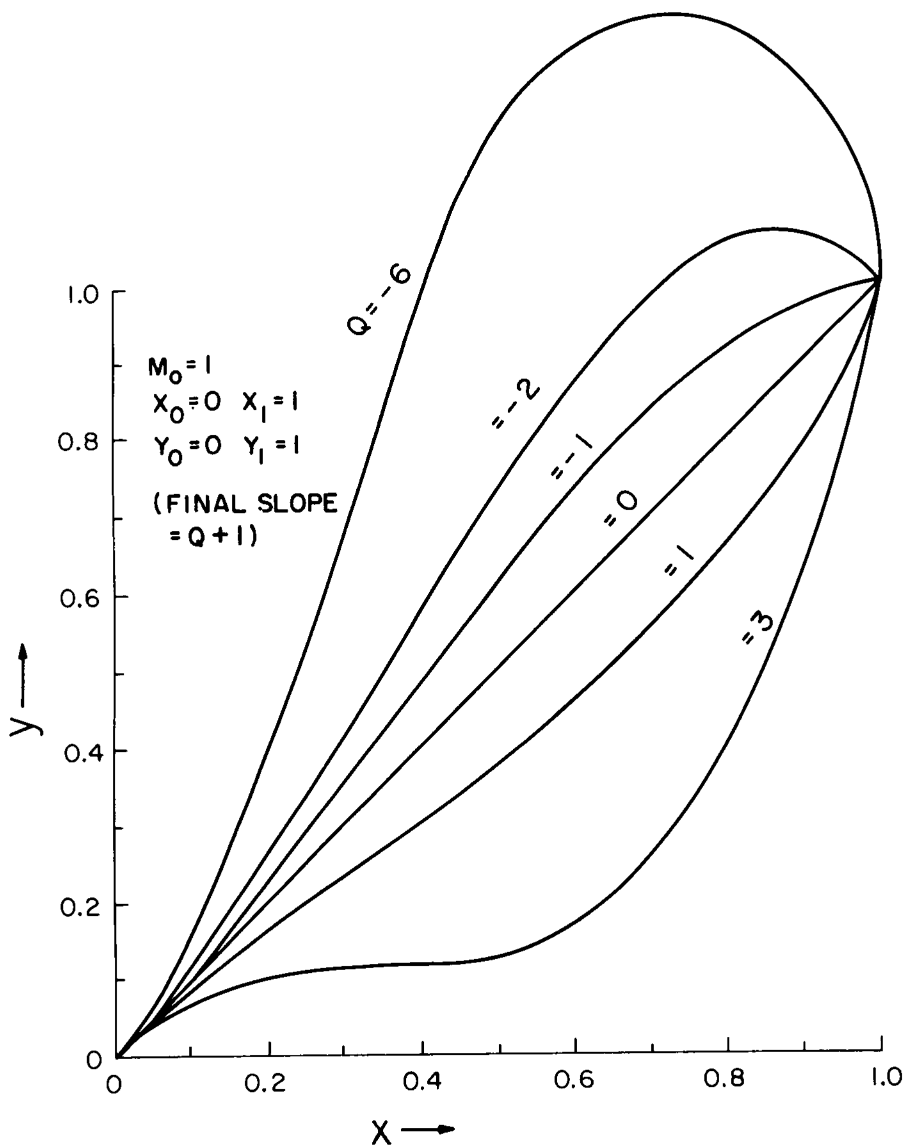


Fig. 3 Curve Segments From Polynomial Realization

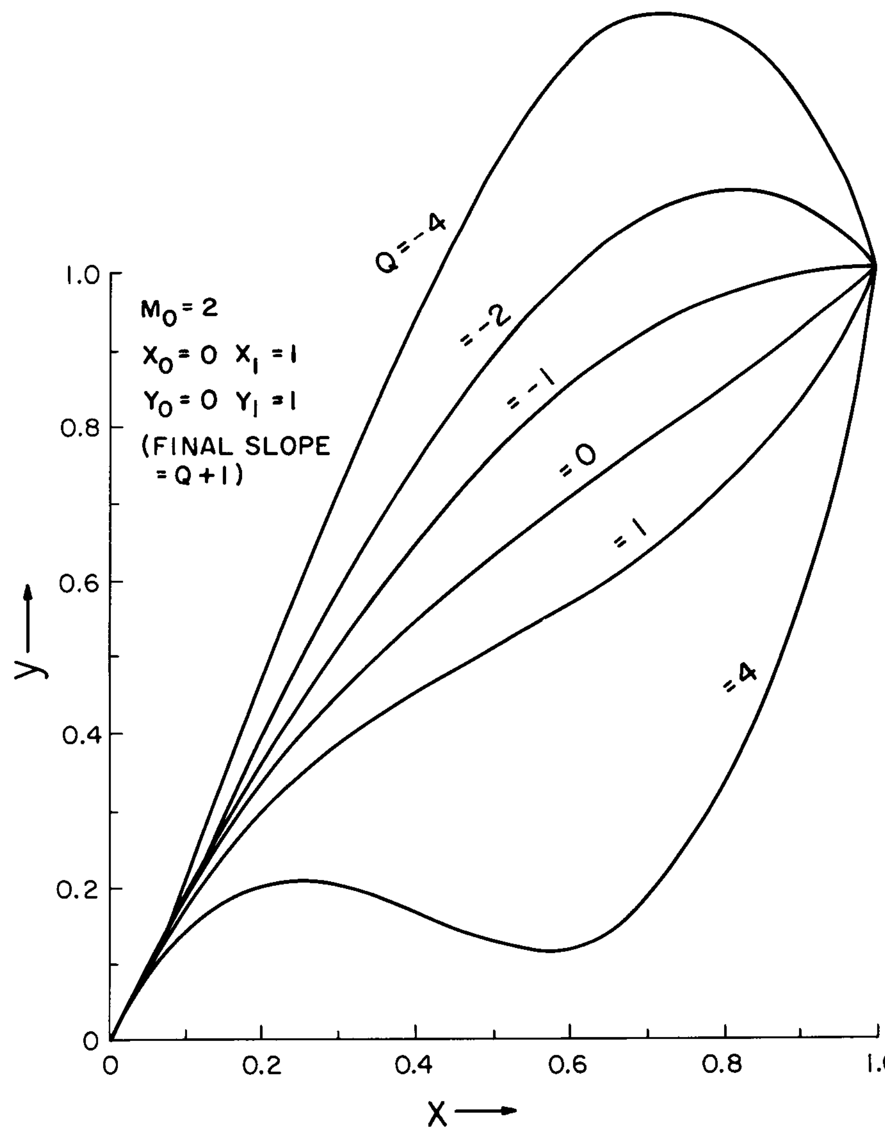


Fig. 4 Curve Segments From Polynomial Realization

form $y(x) = Ax^3+Bx^2+Cx+D$ could be drawn between any two points.

C. EXPONENTIAL REALIZATION

An alternative realization is given by

$$T_x(t) = [1-(a_x e^{-\sigma_0 t} + (1-a_x)e^{-\sigma_x t})]\, U_{-1}(t) \tag{25}$$

$$T_y(t) = [1-(a_y e^{-\sigma_0 t} + (1-a_y)e^{-\sigma_y t})]\, U_{-1}(t) \tag{26}$$

under the constraints

$$0 \leq a_x \leq 1$$

$$0 \leq a_y \leq 1$$

$$a_x < 1 \Rightarrow a_y = 1$$

$$a_y < 1 \Rightarrow a_x = 1$$

$$\sigma_x > \sigma_0 \quad \text{and} \quad \sigma_y > \sigma_0$$

The resulting trajectory from (X_0, Y_0) to (X_1, Y_1) is

$$y = Y_1+(Y_0-Y_1)\left[\frac{a_y(x-X_1)}{(X_0-X_1)} + (1-a_y)\left(\frac{x-X_1}{X_0-X_1}\right)^{\frac{\sigma_y}{\sigma_0}}\right] \quad \text{for} \quad \begin{matrix} a_x = 1 \\ a_y \leq 1 \end{matrix} \tag{27}$$

$$x = X_1+(X_0-X_1)\left[\frac{a_x(y-Y_1)}{(Y_0-Y_1)} + (1-a_x)\left(\frac{y-Y_1}{Y_0-Y_1}\right)^{\frac{\sigma_x}{\sigma_0}}\right] \quad \text{for} \quad \begin{matrix} a_y = 1 \\ a_x \leq 1 \end{matrix} \tag{28}$$

Some typical members of this class are shown in Figs. 5 and 6. The final slope of any such segment, m_f, is

$$m_f = \begin{cases} a_y \dfrac{(Y_0-Y_1)}{(X_0-X_1)} & \text{for} \quad \begin{matrix} a_x = 1 \\ \\ a_y \leq 1 \end{matrix} \\ \\ \dfrac{1}{a_x} \dfrac{(Y_0-Y_1)}{(X_0-X_1)} & \text{for} \quad \begin{matrix} a_y = 1 \\ \\ a_x \leq 1 \end{matrix} \end{cases} \tag{29}$$

From (29), final slope is determined only by parameter a_x or a_y and by coordinate values. Therefore, a large family of curves with a specified slope can be obtained by fixing a_x and a_y and varying the ratio of natural frequencies (as shown in Fig. 6).

Both of the above realizations can display complex curves with smoothly-connected segments. While the first yields a somewhat larger variety of curve segments, it has two distinct disadvantages when compared to the latter. (1) The computing time necessary to select an optimum set of segments to match a given curve is far greater, due to the complexity of the equations relating parameters m_0 and Q to the polynomial coefficients. (2) Implementation is much more complicated and expensive. These disadvantages offset the advantage of a somewhat larger variety of curves and justify implementation of T_x and T_y in the form of Eqs. 25 and 26.

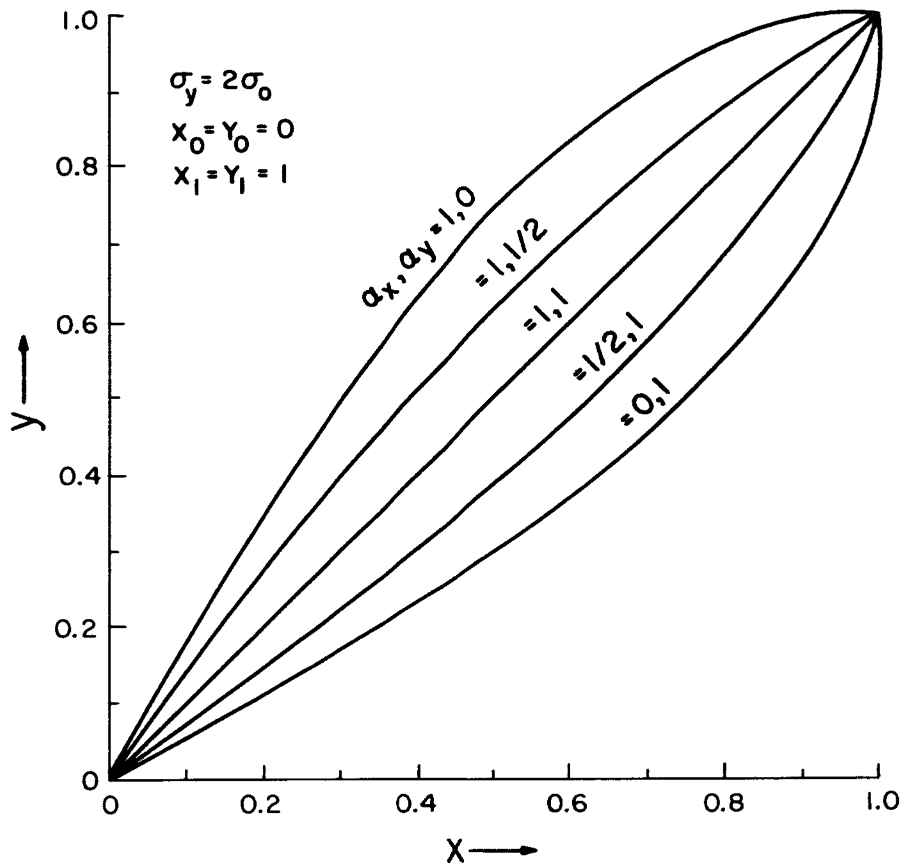


Fig. 5 Curve Segments Obtained from Exponential Realization

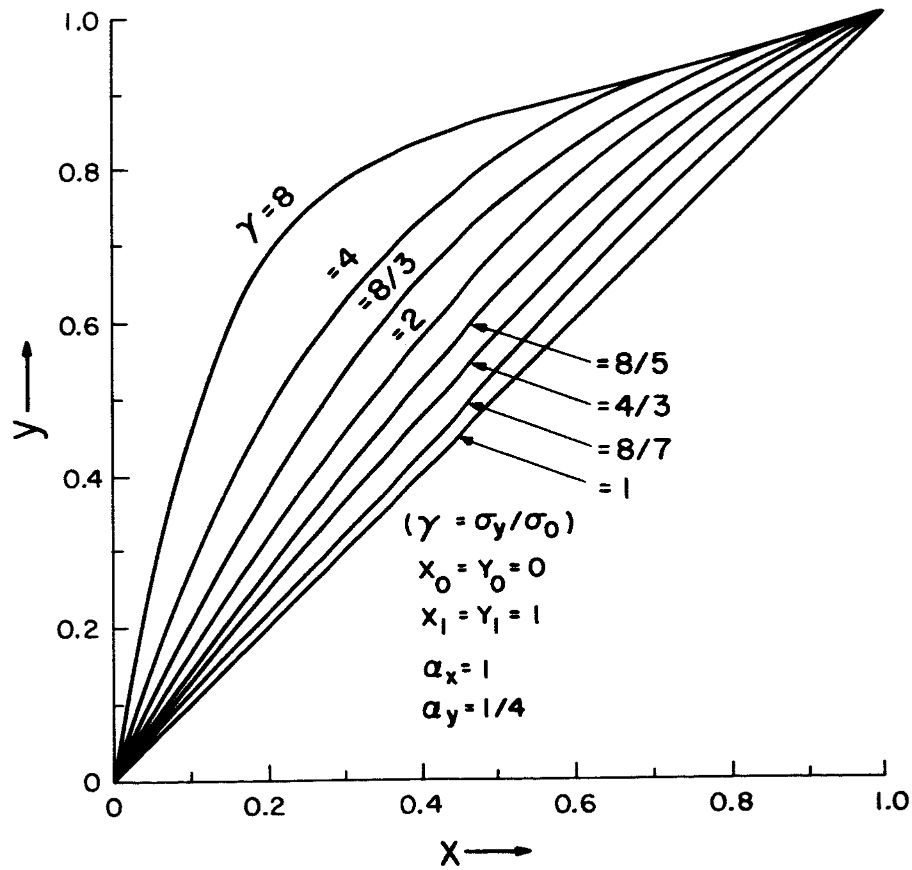


Fig. 6 Curve Segments with the Same Final Slope

D. IMPLEMENTATION

The prototype of this system was designed for use in conjunction with a remote console of the Project MAC[2] time shared system. Digital information is available from the console at a rate of 14 words (four bits per word) per second. Since the display device, a Tektronix type 564 storage oscilloscope, has a resolution of 80 lines per the screen width, six bits are sufficient to specify each coordinate value. Six bits specify a_x and a_y and five bits designate the ratio of natural frequencies (which will be referred to as γ). A single bit is used to specify whether the beam is to be on or off. Therefore 24 bits, or 6 "console words," determine a complete segment. A seventh "console word" is used as a delimiter to separate each segment description.

The complete system has the following basic components:

(1) An interface with the console which

  (a) Converts bit information from the console-to-signal levels compatible with the system logic.

  (b) Generates a strobe pulse, coincident with the console signals, for sampling purposes.

  (c) Recognizes the delimiter signal and generates a delimiter pulse coincident with this signal.

(2) A storage register to store the curve parameters as they arrive from the console.

(3) A word designator which "points" to the portion of the storage register into which incoming "console word" is stored. The

designator is updated by each strobe pulse and reset by each delimiter pulse.

(4) Two D/A converters, and associated holding registers (which are loaded by the delimiter pulses), to convert the present coordinate values to analog form.

(5) An intensity control which increases the beam intensity when a segment is plotted.

(6) T_x and T_y.

These basic components are shown in the block diagram of Fig. 7.

E. IMPLEMENTATION OF T_x AND T_y

The design of most of these components is quite straightforward as seen from their schematics in the appendix. What is of primary interest is the design of T_x and T_y.

Equations 25 and 26 specify the configuration shown in Fig. 8. This configuration can be reduced by sharing components between T_x and T_y. Note that, for any given segment, only one α multiplier, one $(1-\alpha)$ multiplier, and one variable resistor is used. Sharing these three components reduces T_x and T_y to the configuration shown in Fig. 9. The switch control, α_s, is one of the six bits which specify α. It differentiates between the two cases: $(\alpha_x = 1,\ \alpha_y \leq 1)$ and $(\alpha_y = 1,\ \alpha_x \leq 1)$.

The significant criteria which must be met by the design of this network are:

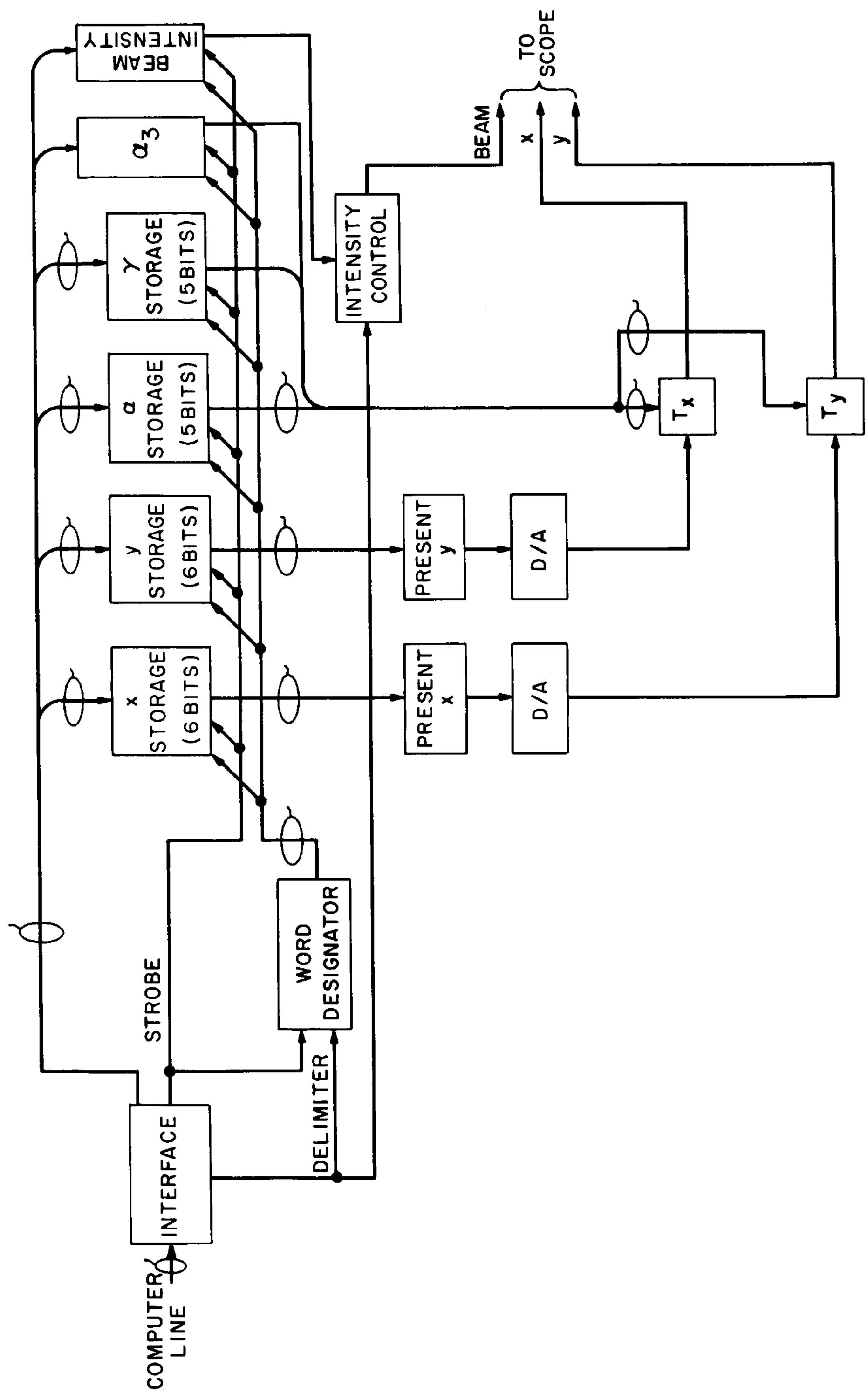


Fig. 7 Block Diagram of Complete System

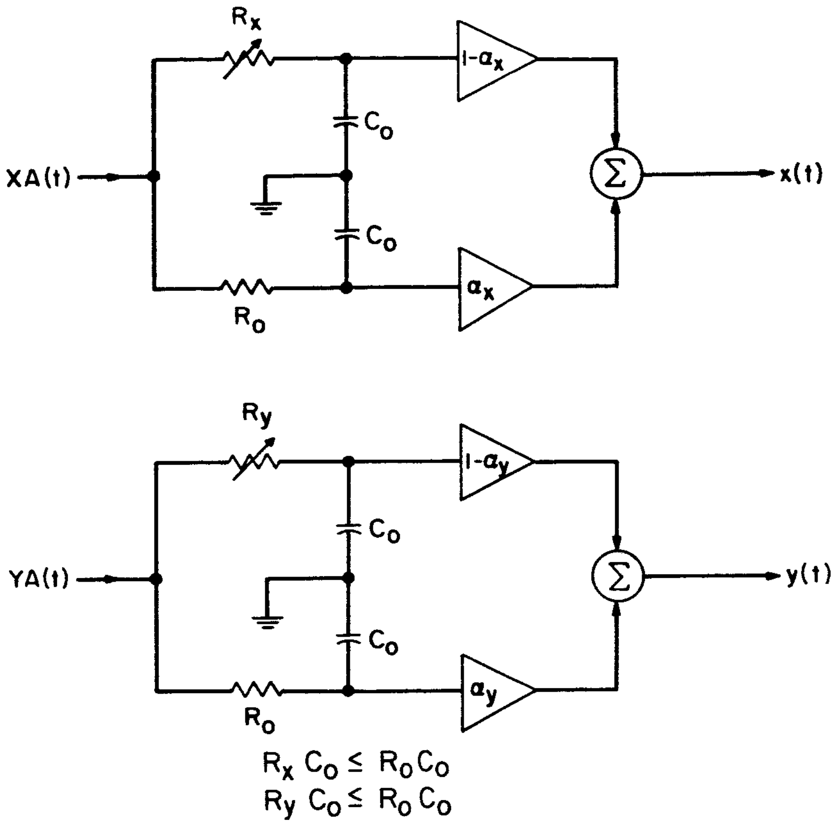


Fig. 8 Configuration of T_x and T_y

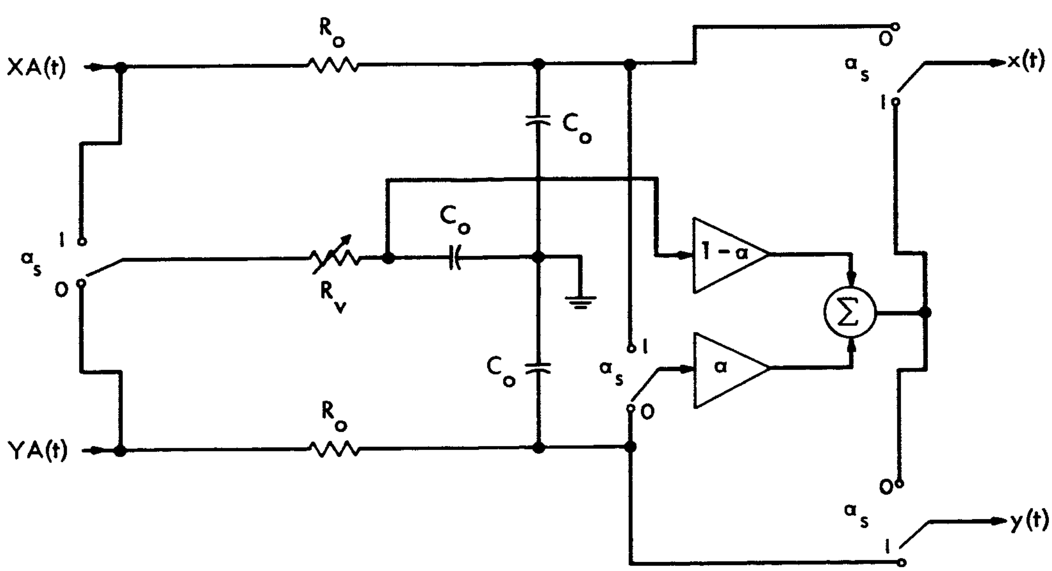


Fig. 9 Reduced Configuration of T_x and T_y

(1) The network must reach steady state within 200 Msec so that subsequent changes in R_v and a will not effect the plotted segment.

(2) The time constants of the network must be long enough so that the finite rise time of the step inputs will have negligible effect on x(t) and y(t).

(3) The accuracy of the multipliers must be one percent or better to match the accuracy of the coordinate values.

(4) R_v must be varied in such a way as to produce a well-spaced family of curves.

Conditions (1) and (2) were easily met. The input rise time is on the order of 1 μsec leaving a large range in which to set the basic time constant of the network. This time constant, R_0C_0, was chosen to be 4 Msec which is small enough to allow the system to be used with a much higher data rate channel if so desired.

The multipliers were designed in the form of the resistive divider network shown in Fig. 10. Assume, for the moment, $a_0 = 0$. Then V_{out}/V_{in} will be

$$\frac{V_{out}}{V_{in}} = \frac{a_1 2^{n-1} + a_2 2^{n-2} + \ldots + 2a_{n-1} + a_n}{2^n} \tag{30}$$

where the a_j have value 0 or 1. Therefore, if the value of a is coded with $n+1$ bits, V_{out}/V_{in} can be varied from zero to one in steps of $1/2^n$. The $(1-a)$ multiplier is obtained by reversing

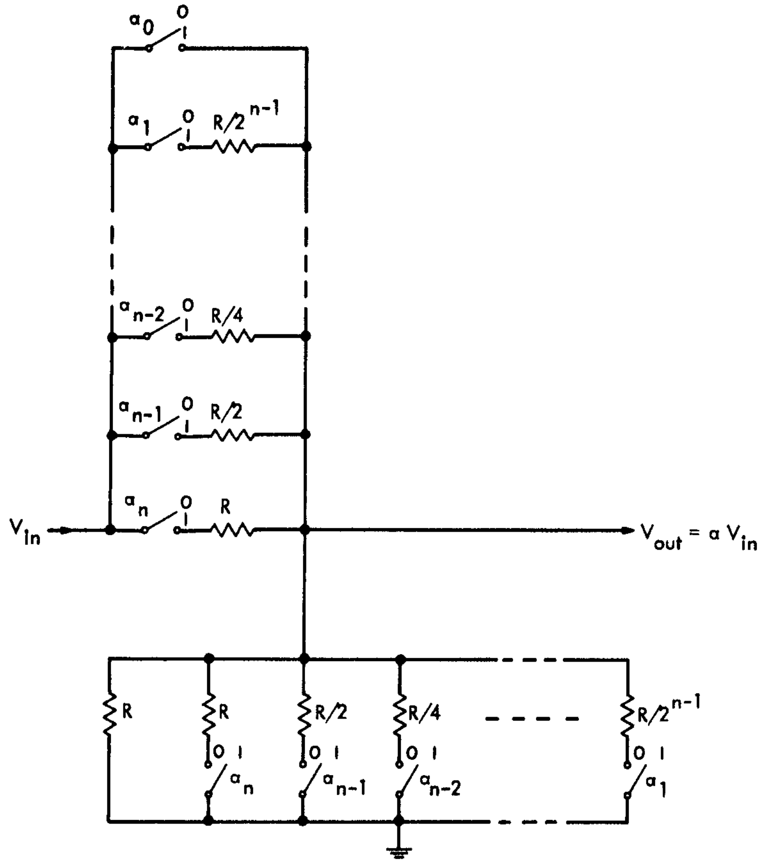


Fig. 10 Configuration of the a Multiplier

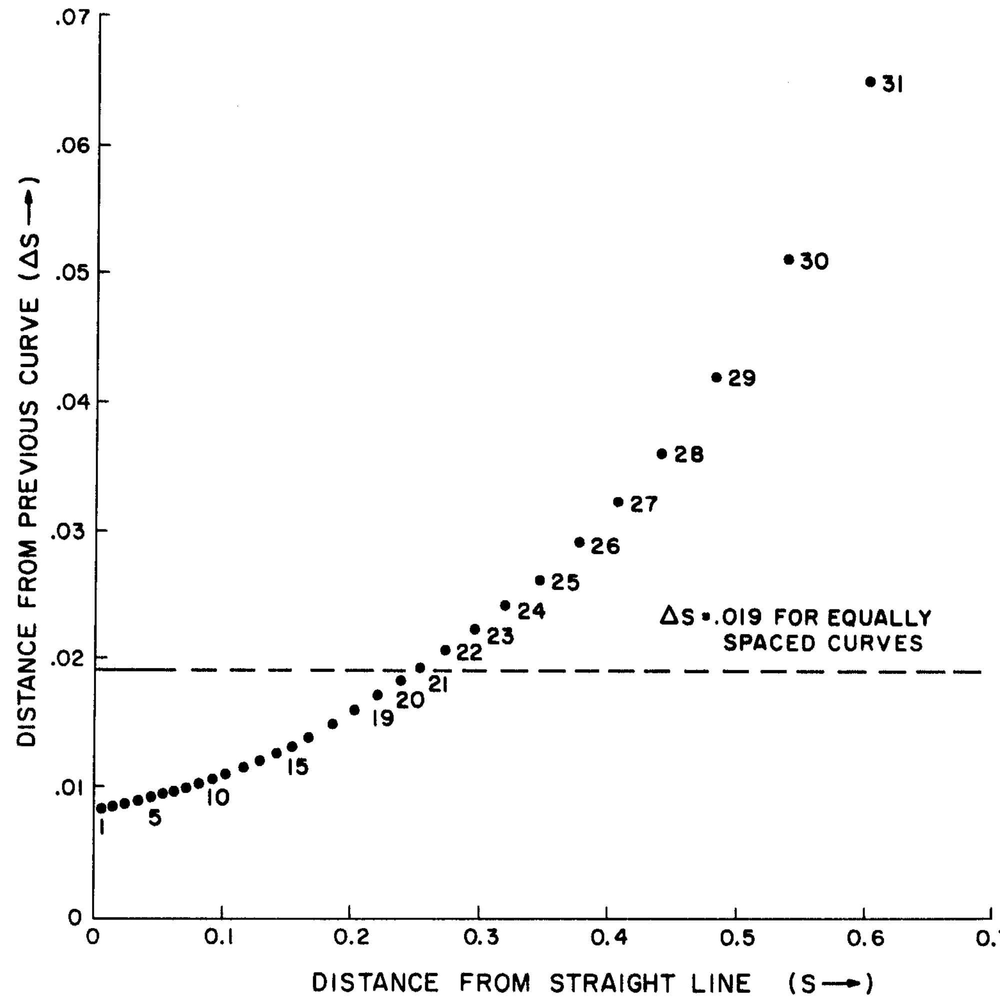


Fig. 11 Spacing of Curve Segments

the input and datum nodes. To keep the accuracy of the multipliers within one percent, the tolerance of the resistors must be 1/2 percent. This tolerance is achieved with trimmer resistors in the prototype. A final version of the system should use precision resistors to eliminate repeated calibrations. A complete schematic of the multiplier is shown in the appendix.

Five bits are used to specify the ratio of natural frequencies (γ). Since segments with a small, slowly changing curvature will be used more often than segments with large, rapidly changing curvature, it is desirable to have the segments closely spaced for small values of γ with the spacing increasing with increasing values of γ. A distribution such as this can be obtained by increasing γ in steps of $\Delta\gamma$ given by

$$\Delta\gamma = \frac{\gamma^2}{32-\gamma} \quad . \tag{31}$$

The spacing obtained from (31) is plotted in Fig. 11 for the set of curves between points (0,0) and (1,1). The distance between the centers of successive segments is ΔS and the distance from the connecting line is S. Note that ΔS gradually increases with S as desired. Equation 31 is realized by the configuration of Fig. 12. With this configuration R_v is given by

$$R_v = \frac{R_0}{32}\,[16\gamma_1 + 8\gamma_2 + 4\gamma_3 + 2\gamma_2 + \gamma_1] \quad . \tag{32}$$

resulting in

$$\gamma = \frac{32}{16\gamma_1 + 8\gamma_2 + 4\gamma_3 + 2\gamma_4 + \gamma_5} . \tag{33}$$

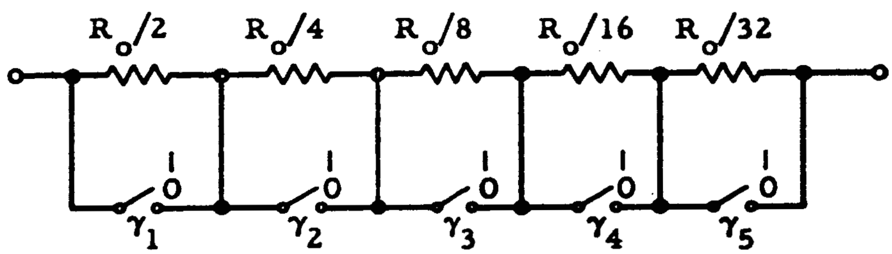


Fig. 12 Configuration of R_v

A change in the least significant bit designating γ results in the $\Delta\gamma$ given by (31).

Magnetic reed switches were used in the implementation of R_v and elsewhere whenever a floating switch was needed. These switches have a 1 Msec switching time which is adequate for the present system. Solid-state switches can be substituted if the plotter were to be used with a higher data-rate channel.

Schematics of T_x and T_y and other major components of the system are given in the appendix.

IV. SOFTWARE CONSIDERATIONS

The previous sections have described a method for specifying and displaying complex curves with a small set of computer words. These words consist of coordinate values and the specification of connecting curve segments. The problem remains to develop an algorithm for choosing, from the available family of segments, an optimum set which will match a given curve within some performance criteria.

The algorithm chosen will depend upon the class of graphical data to be displayed. Tradeoffs exist between computing time, complexity of the software, and the quality of display desired. To test and evaluate the display technique, the chosen criteria matched the curve with the minimum number of segments possible within a maximum specified error. Where possible, and desirable, the slopes of connecting segments were matched. A brief flow-chart describing this algorithm is shown in Fig. 13. It has been assumed that the curve to be displayed is stored within the computer as an ordered set of closely spaced points, specified by their coordinate values.

As shown, the algorithm is divided into three major sections. (1) The curve is scanned to locate all local maximum and minimum points in both the x and y directions. Since all segments are single-valued in both x and y, no single curve can contain such a point. Thus the curve is matched between successive maximum-minimum points with a minimum number of segments. This set of points and the initial and final point are stored in the array P,

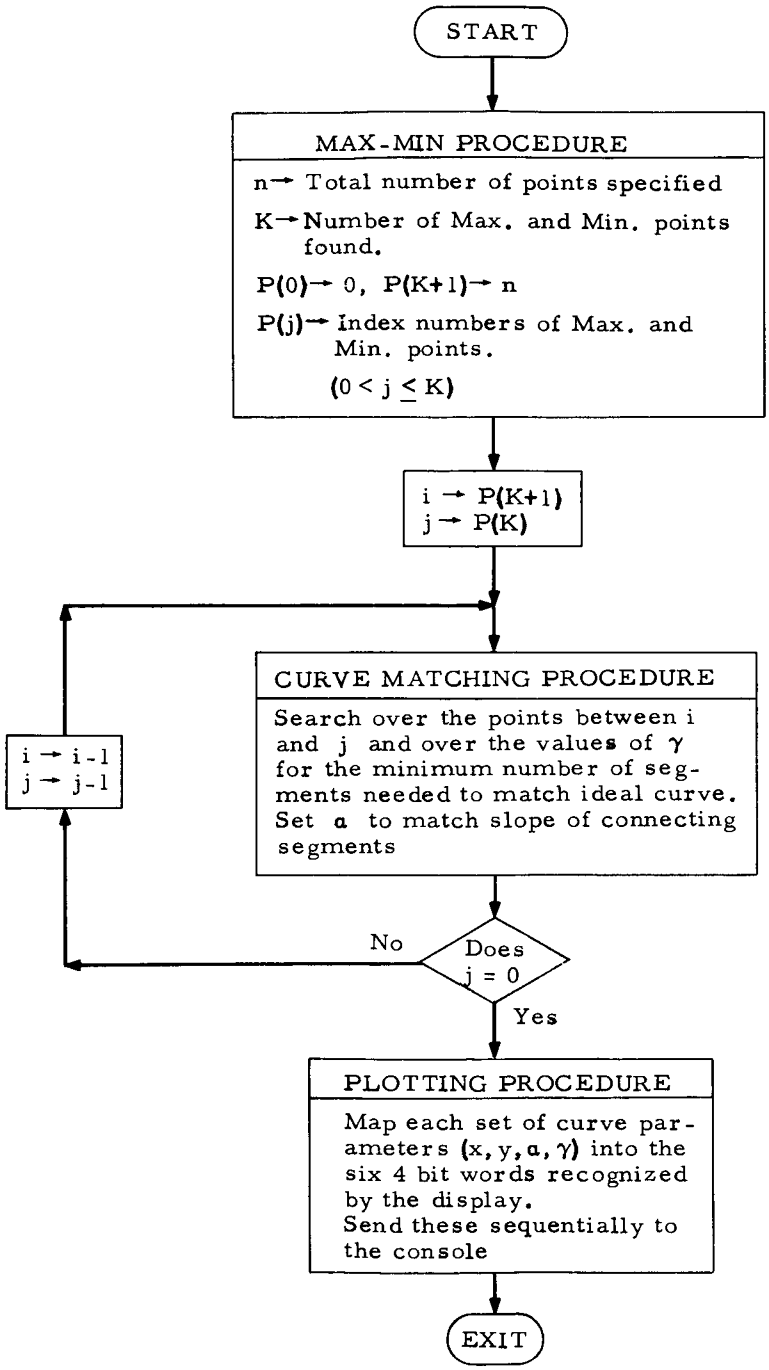


Fig. 13 Flow Chart of Curve Matching Algorithm

as shown. (2) Next, a search is conducted, between successive point pairs in P, for the minimum number of curve segments that match the ideal curve within the specified error. An exponential search is used to minimize computer time. For each segment, a is set to match the slope of the previous segment or, if this is not possible, to match the slope of the given curve. (3) Finally, the computer words found in step 2 are mapped into the corresponding symbols to be set to the console.

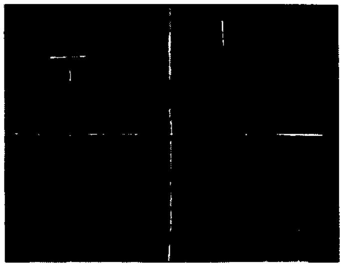
Detailed flow charts of the significant portions of the algorithm are included in the appendix. The program for this procedure was written in the AED version of Algol as used by Project MAC.

V. EXPERIMENTAL RESULTS AND CONCLUSIONS

The prototype hardware and translator software were used in conjunction with CIRCAL,[3] a program for online analysis of electronic circuits. The results of this test are shown in Figs. 14, 16, 17, and 18. Figure 14 shows the output voltage vs. time for the circuit of Fig. 15. This curve, originally specified by CIRCAL at 100 points, was drawn with four curve segments. The ideal curve was matched within 0.5 percent using 0.5 sec. of computer time. Increasing the allowable error to 3 percent gave rise to the curve in Fig. 16. This fit was accomplished with three segments using 0.3 seconds of computer time. Figure 17 shows the responses of this circuit before and after the capacitance is increased to 0.1. The stored display allows curves to be compared in this manner. Figure 18, the response to the circuit of Fig. 19, was plotted with 8 segments within an error of one percent. This curve was also originally specified as 100 closely spaced points.

The system can, of course, be used to display any class of graphical data. For instance, the car in Fig. 20, supplied to the translator as 200 points, was plotted with 26 segments. The translator time in this case was 2.8 seconds.

The above results demonstrate one major feature of this device. Relatively few computer words are needed to specify a relatively complex curve. Two significant consequences of this feature are: (1) Storage requirements for graphical data are



Horizontal Scale:.25 sec/div
Vertical Scale: .4 volts/div

Fig. 14 Voltage Response of Tunnel Diode Circuit (0.5 Percent Fit)

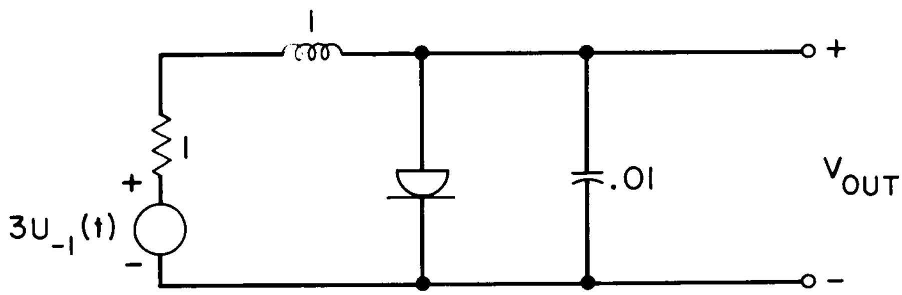
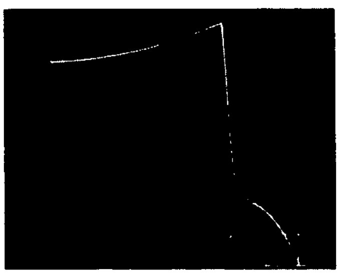
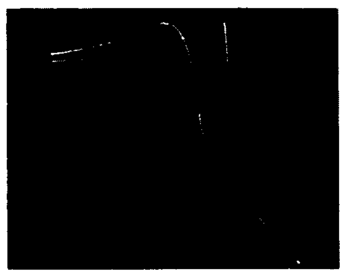


Fig. 15 Tunnel Diode Circuit



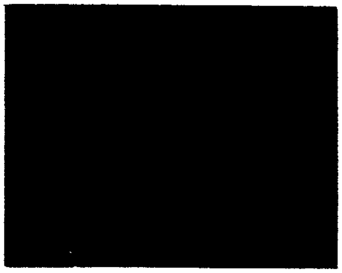
Horizontal Scale: .25 sec/div
Vertical Scale: .4 volts/div

Fig. 16 Voltage Response of Tunnel Diode Circuit (3 Percent Fit)



Horizontal Scale: .25 sec/div
Vertical Scale: .4 volts/div

Fig. 17 Superimposed Voltage Responses



Horizontal Scale: 1.6 sec/div
Vertical Scale: .04 volts/div

Fig. 18 Step Response of a Damped L-C Circuit

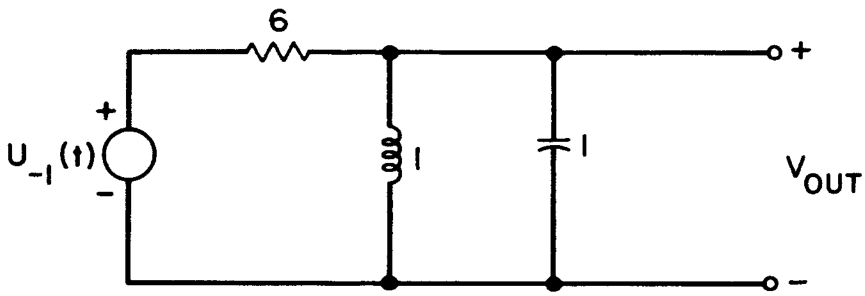


Fig. 19 Damped L-C Circuit

Fig. 20 Automobile Silhouette

decreased. (2) Plotting speed is considerably higher than that of a conventional point-by-point display operating from the same data-rate channel. A third advantage results from the simplicity of the plotting technique. The basic circuits used are quite straightforward, therefore reliable and inexpensive.

The basic drawback is the computer time necessary to translate a curve into a segment description.

It appears then that the primary value of this technique may be in the storage and display of curves which will be translated once and referenced many times. However, in many cases, this translation time may be insignificant. For instance, when the display is used with CIRCAL, translation time is a fraction of a second, while actual analysis of the circuit takes several seconds. For cases where translation time is undesirable, the device has the ability to draw straight-line segments between any two grid points (by setting a to one); translation time can thus be eliminated.

APPENDIX

Figures A through F of the Appendix are detailed schematic diagrams of the major components shown in Fig. 7 of the text. Figures G, H, and I consist of detailed flow diagrams of the computer software described in Chapter IV.

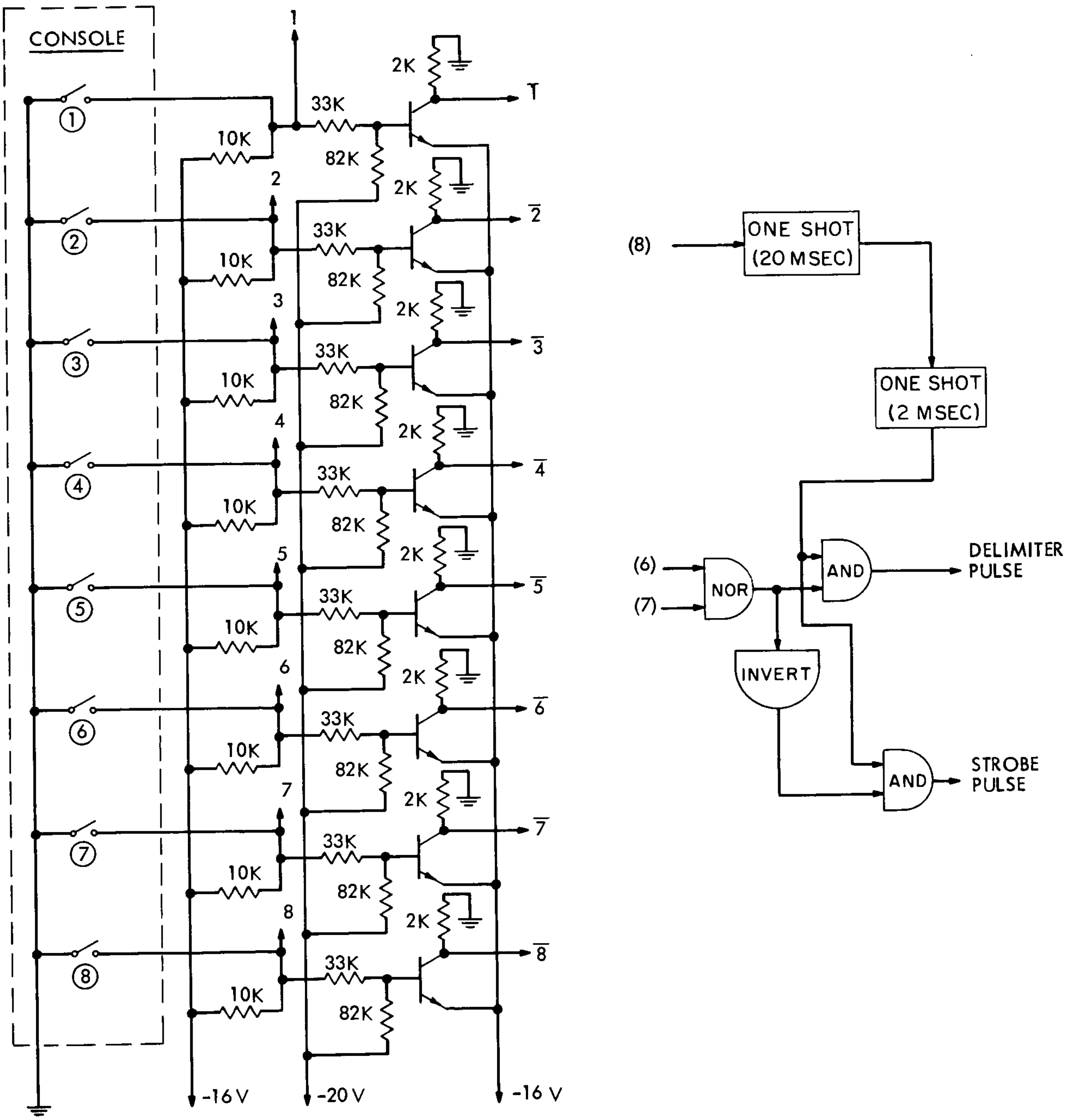


Fig. A Interface Circuit

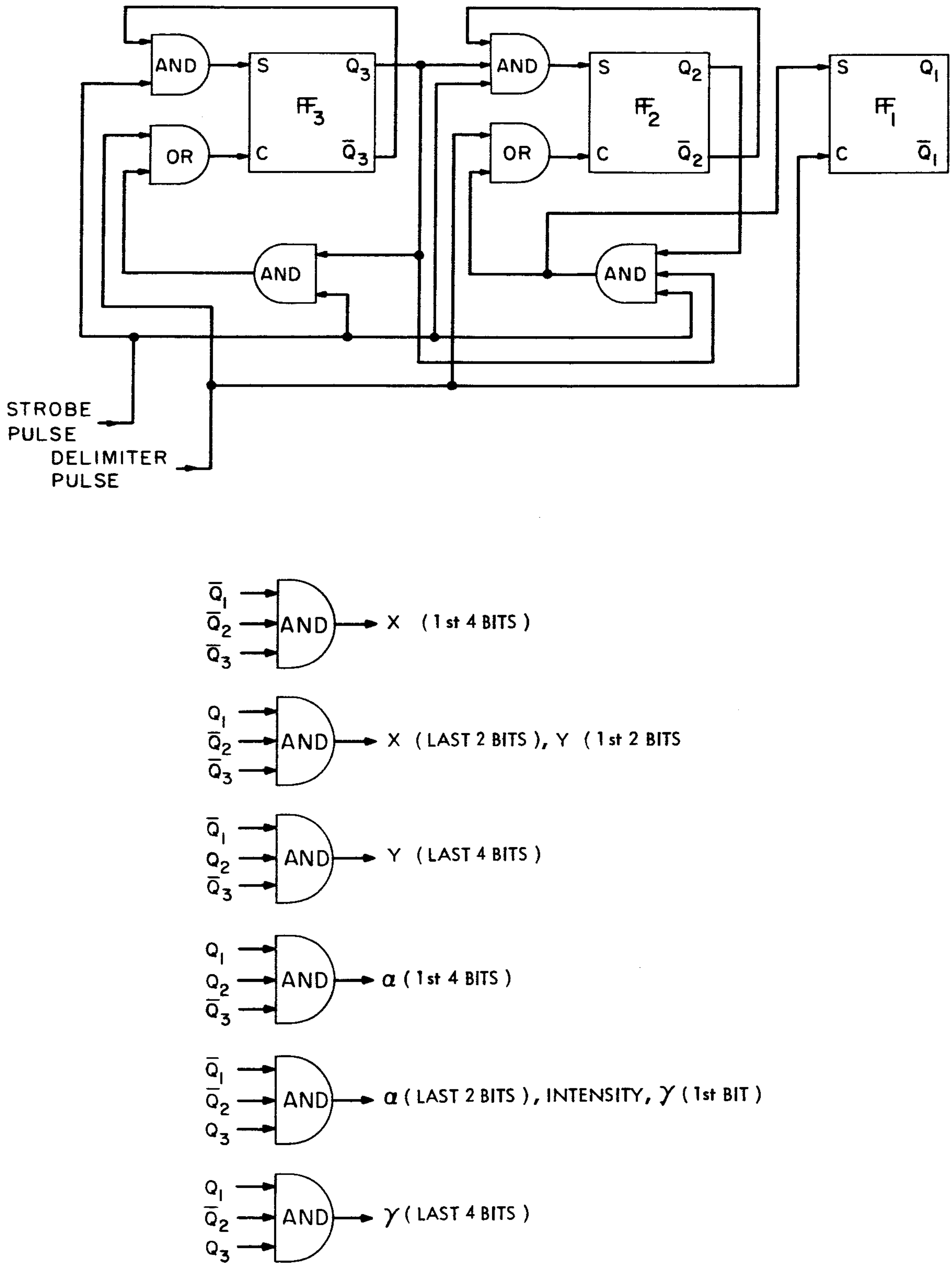


Fig. B Word Designator

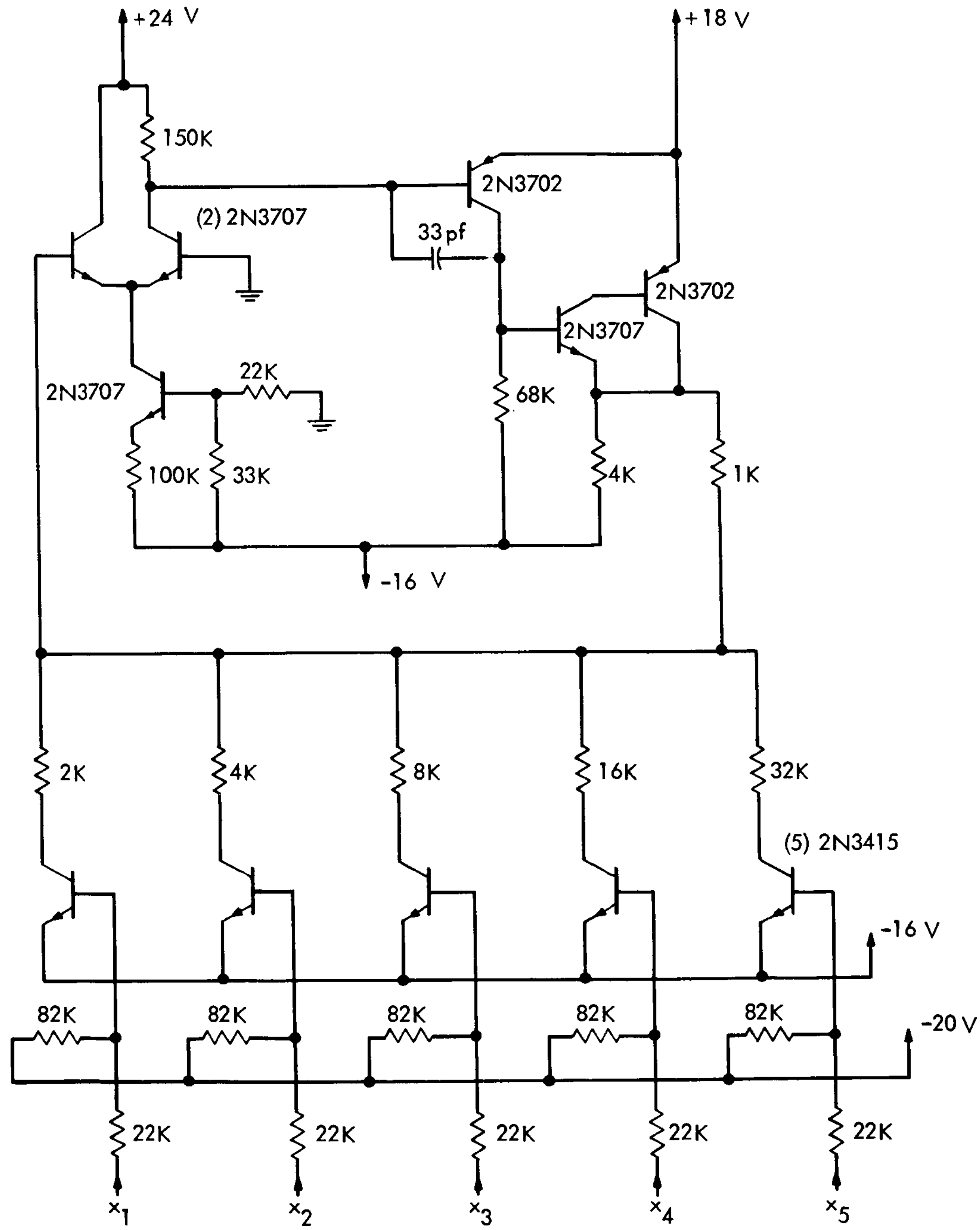


Fig. C Digital-to-Analog Converter

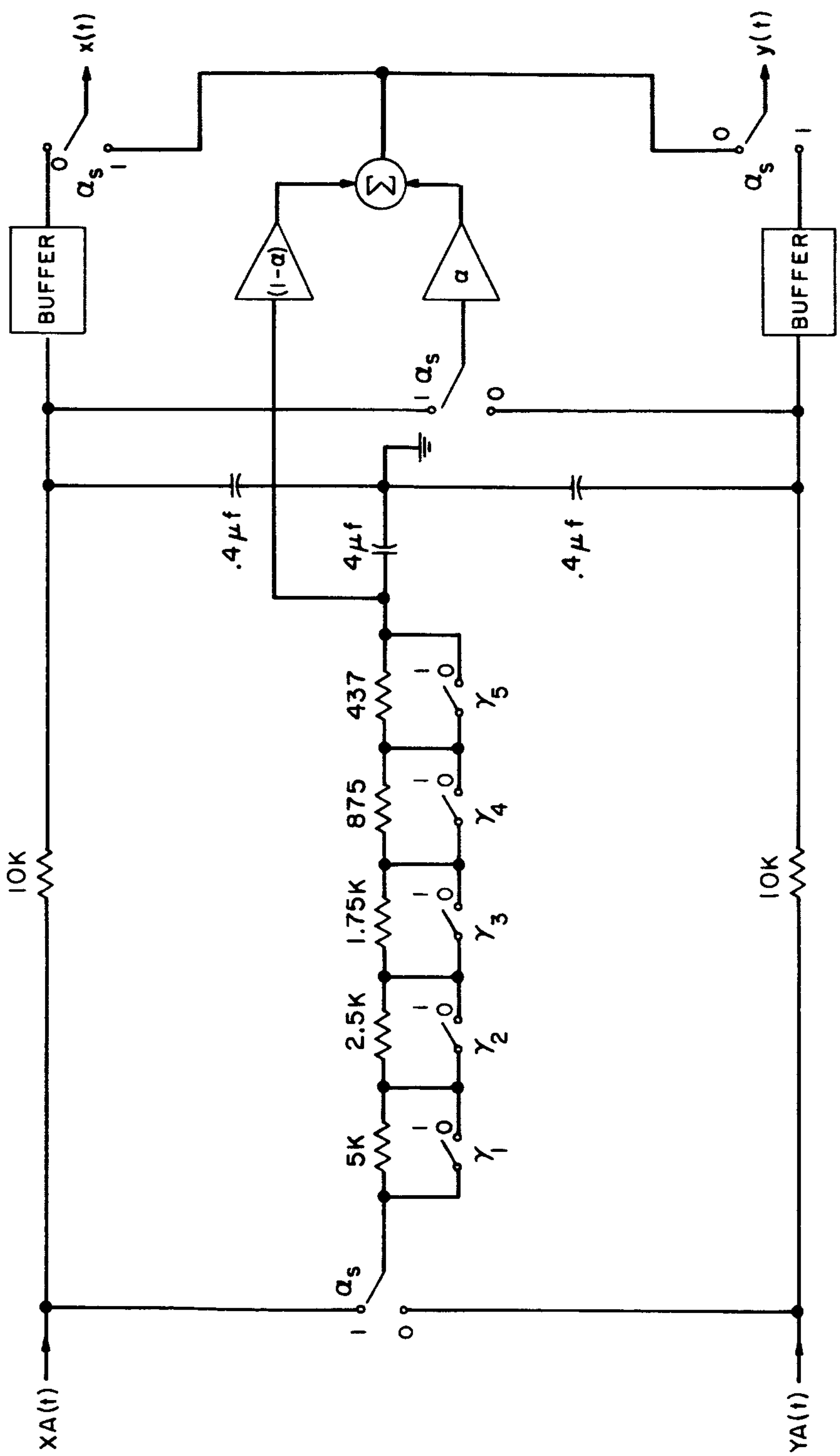


Fig. D Circuit of T_x and T_y

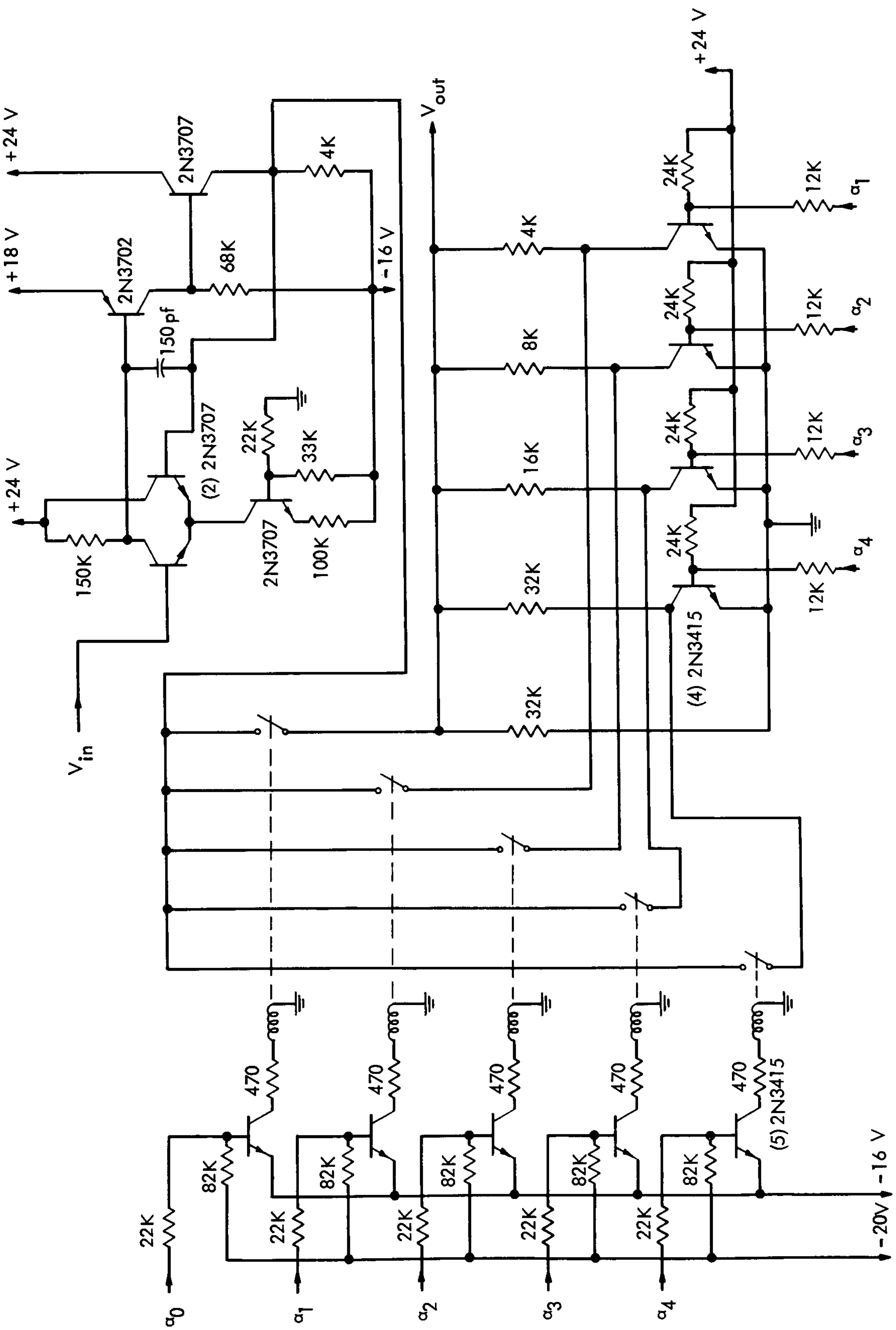


Fig. E α Multiplier Circuit

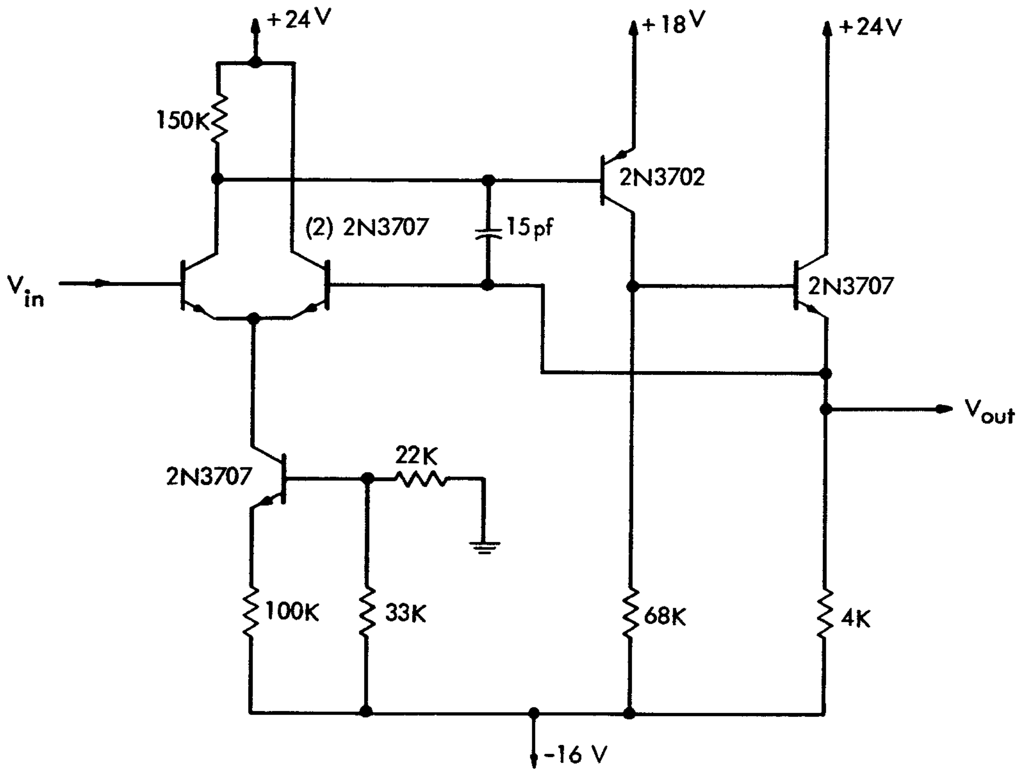


Fig. F Unity Gain Buffer

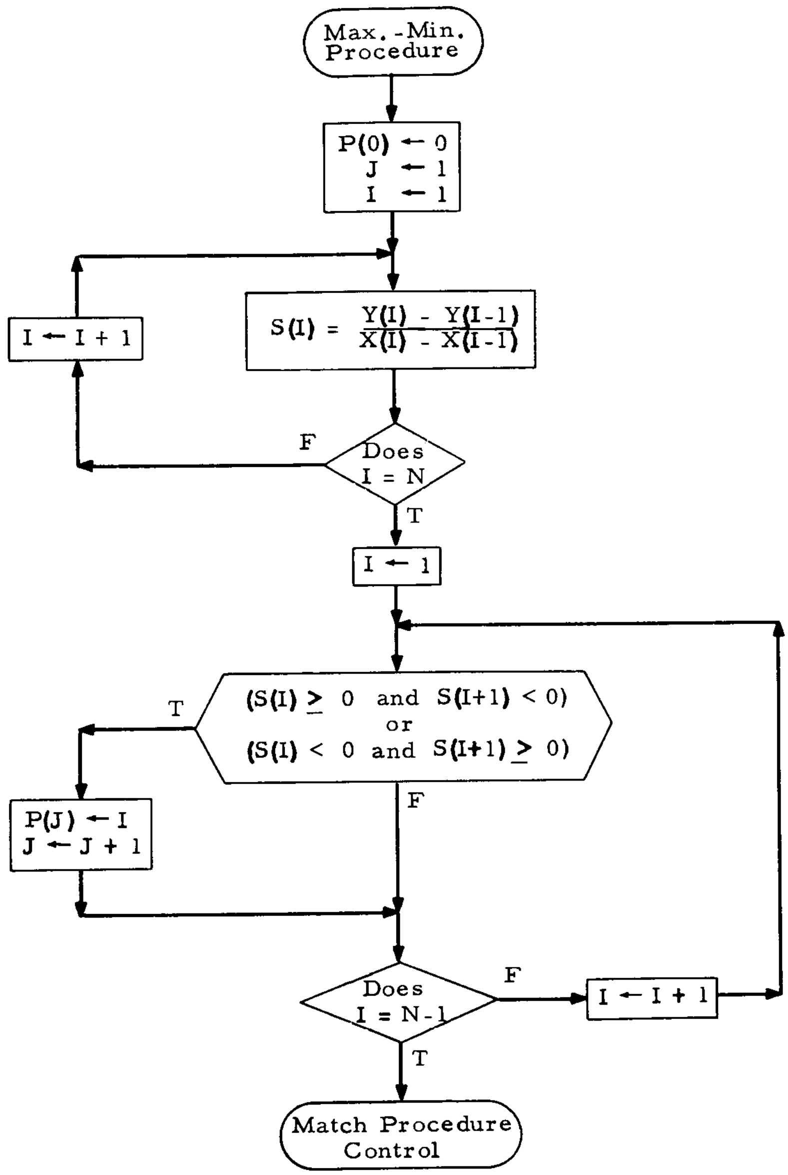


Fig. G Maximum-Minimum Procedure

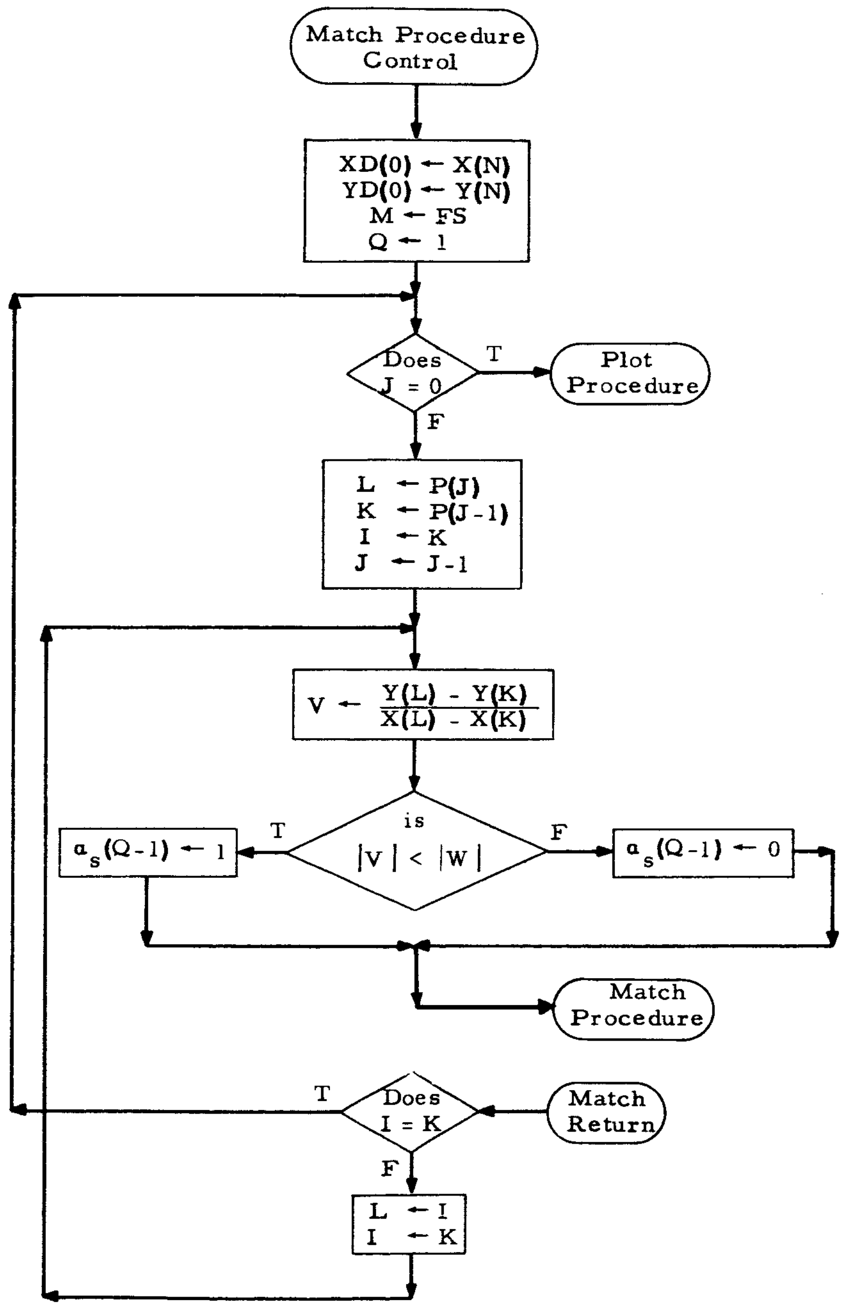


Fig H Match Procedure Control

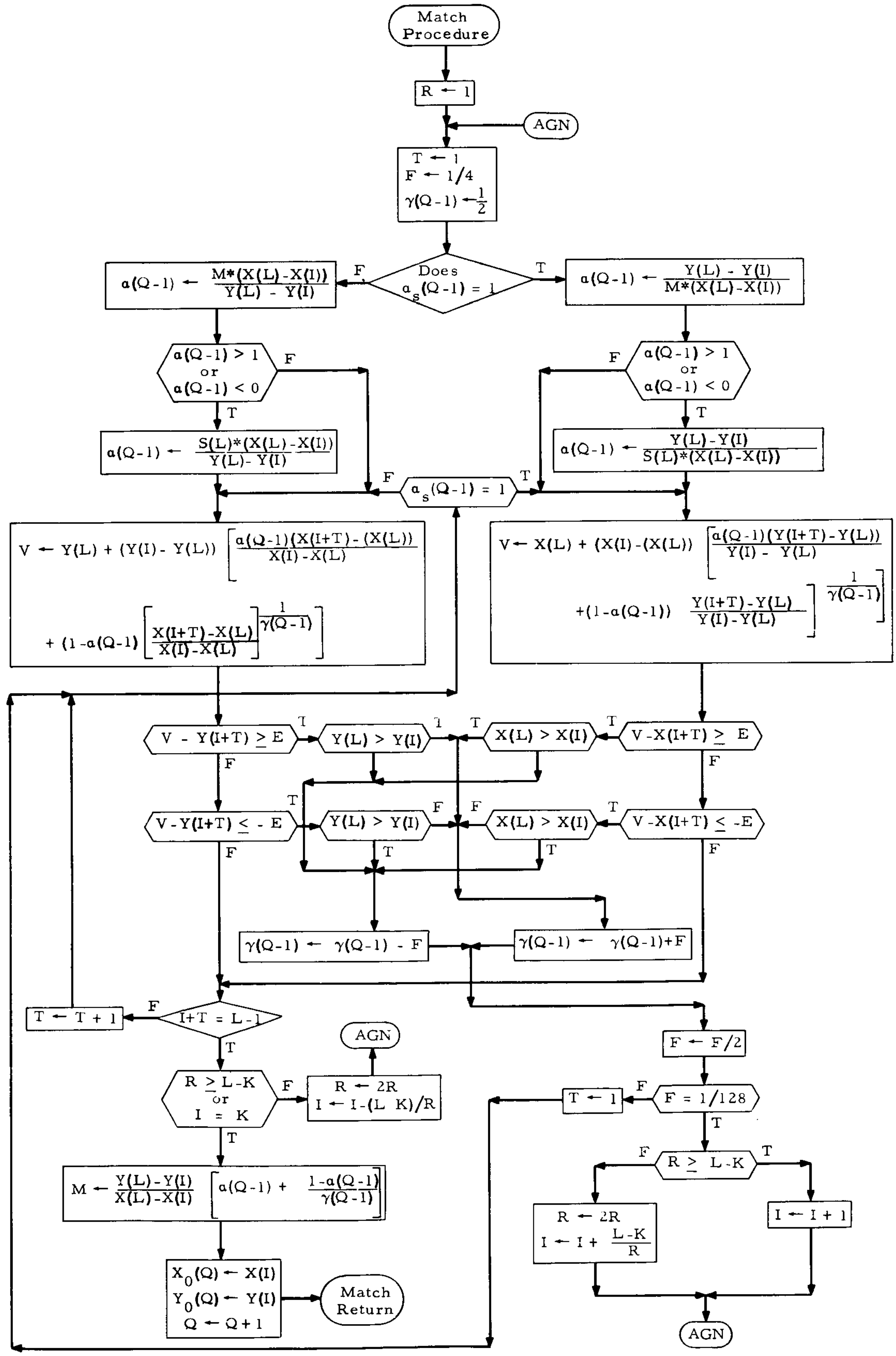


Fig. 1 Match Procedure

BIBLIOGRAPHY

1. Dertouzos, M. L., "Phaseplot: A Hybrid Graphical Display Technique for Digital Computer Use," September, 1965, Unpublished.

2. Massachusetts Institute of Technology Project MAC Progress Report, July, 1964.

3. Dertouzos, M. L. and Therrien, C. W., "CIRCAL: On-Line Analysis of Electronic Networks," Report ESL-R-248, Electronic Systems Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts, October, 1965.